Discount Rates in Financial Reporting: A Practical Guide

Extrapolation of yield curve, credit and liquidity risk, inflation

Jeremy Kent
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Zurich
Extrapolation of yield curve

- Sometimes need to project liability cashflows extending beyond the longest maturity asset yield observable in the market.
- Yields can include those on derivatives (e.g. swaps). Swap rates may be observable and liquid beyond bond maturities.
- Solvency II refers to “deep, liquid and transparent markets”
Extrapolation of yield curve – five example methods

• Based on spot rates
• Based on forward rates
• Price-consistent extrapolation
• Solvency II method
• Extrapolation based on comparable currencies
Extrapolation of yield curve – extrapolation based on spot rates

- Assumes longest market-observed spot rate continues thereafter
- Example below assumes longest market-observed spot rate is 10-years:

- Method assumes constant forward rates in extrapolated period => discontinuities in forward rates
Extrapolation of yield curve – extrapolation based on forward rates

- Assumes longest market-observed forward rate continues thereafter
- Example below assumes longest market-observed forward rate is 10-years:

- Constant forward rates => spot rates continue to have positive slope (assuming yield curve not inverted)
- Fewer (or no) discontinuities
Extrapolation of yield curve – extrapolation based on forward rates (2)

• Very sensitive to forward rates/spot yields observed in later years (especially an issue if volatile due to supply/demand etc)

• Moving-average forward yields can be used to reduce volatility, but may reduce market consistency
Extrapolation of yield curve – price-consistent extrapolation

• Market-consistent yield curve constructed based on market-traded asset whose cash-flows extend as long as the projected liabilities
• Risk/return profile of assets need to be reasonably similar to profile of liability cash-flows
• Asset should be traded in reasonably liquid market between arm’s length participants
• Find very long maturity yields which, when appended to market-observed yields, produce discounted asset cash-flows equal to market value => market consistency
Extrapolation of yield curve – Solvency II extrapolation method

- Final methodologies still under development
- Basic risk-free interest rates derived from interest rate swaps for the currency, adjusted for credit risk
- For currencies where swap rates not available from deep, liquid and transparent markets, yields on government bonds (adjusted for credit risk) used, where available from deep, liquid and transparent markets
- SII uses macroeconomic method:
  - Reflects current market conditions
  - Embodies economic views on how unobservable long term rates expected to behave
  - Assumes long term equilibrium interest rate
  - Ensures relatively stable results over long term
Extrapolation of yield curve – Solvency II extrapolation method (LLP)

- Extrapolation made from last liquid point (LLP)
- Minimum of:
  - Highest maturity for which reference instrument markets (e.g. swaps) are active, deep, liquid, transparent
  - Highest maturity where overall bond market is ADLT (sovereigns and corporates)
- For bond market LLP also reflects insurers ability to match liabilities with bonds
Extrapolation of yield curve – Solvency II extrapolation method (UFR)

- Solvency II forward rates converge to macro-economically assessed ultimate forward rate (UFR)
- Revised regularly, but expected to be stable over time
- Key factors:
  - Long-term expected inflation
  - Expected real interest rates.
- Convergence reached when within 3bp of UFR
Extrapolation of yield curve – Solvency II extrapolation method (UFR (2))

• Long term inflation
  – EIOPA considered inflation from 1994-2010 for OECD and some other Asian countries
  – Inflation more modest in last 15-20 years than previously; many central banks have set inflation targets and been successful in controlling inflation
  – EIOPA proposed 2%
Extrapolation of yield curve – Solvency II extrapolation method (UFR (3))

• For 19 economies, 1900-2010:

Figure 1: Real return on bonds 1900 – 2010
Source: Dimson, Marsh and Staunton – Credit Suisse Global Investment Returns Yearbook 2011

Real return on bonds 1900-2010
Extrapolation of yield curve – Solvency II extrapolation method (UFR (4))

- For 12 economies:

*Figure 2: Real bond returns: first versus second half of 20th century*
Source: Dimson, Marsh and Staunton (ABN-Amro/London Business School)

* Data for Germany excludes 1922-23. AVG = Average
Extrapolation of yield curve – Solvency II extrapolation method (UFR (5))

- Expected real interest rates
  - real bond rates 2.3% over second half of 20th century
  - -1.1% over first half (impact of high or hyperinflation)
  - EIOPA proposed 2.2%
Extrapolation of yield curve – Solvency II extrapolation method (parameters)

Some example parameters

<table>
<thead>
<tr>
<th>Currency</th>
<th>LLP (years)</th>
<th>Convergence (years after LLP)</th>
<th>UFR %</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro</td>
<td>20</td>
<td>40</td>
<td>4.2</td>
<td>Swaps</td>
</tr>
<tr>
<td>GBP</td>
<td>50</td>
<td>40</td>
<td>4.2</td>
<td>Swaps</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>20</td>
<td>40</td>
<td>3.2</td>
<td>Swaps</td>
</tr>
<tr>
<td>Swedish Krona</td>
<td>10</td>
<td>10</td>
<td>4.2</td>
<td>Swaps</td>
</tr>
<tr>
<td>Croatian Kuna</td>
<td>7</td>
<td>40</td>
<td>4.2</td>
<td>Govt bonds</td>
</tr>
<tr>
<td>Polish Zloty</td>
<td>15</td>
<td>40</td>
<td>4.2</td>
<td>Govt bonds</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>25</td>
<td>35</td>
<td>3.2</td>
<td>Swaps</td>
</tr>
</tbody>
</table>
Extrapolation of yield curve – Solvency II extrapolation method (Smith-Wilson method)

• Input data can consist of different financial instruments; simpler case assumes zero coupon bond prices
• Assume risk-free zero coupon rates known for N maturities \( u_1, u_2, u_3, \text{ up to } u_N \)
• Input ZC bond prices (paying 1):

\[
m_i = P(u_i) = \exp(-u_i \cdot \tilde{R}_{u_i}) \quad \text{for continuously compounded rates, and}
\]

\[
m_i = P(u_i) = (1 + R_{u_i})^{-u_i} \quad \text{for annual compounding.}
\]

• Smith and Wilson pricing function:

\[
P(t) = e^{-UFR \cdot t} + \sum_{j=1}^{N} \zeta_j \cdot W(t, u_j), \quad t \geq 0
\]
Extrapolation of yield curve – Solvency II extrapolation method
(Smith-Wilson method (2))

• Symmetric Wilson functions:

\[ W(t, u_j) = e^{\text{UFR}(t+u_j)} \left\{ \alpha \cdot \min(t, u_j) - 0.5 \cdot e^{-\alpha \cdot \max(t, u_j)} \cdot (e^{\alpha \cdot \min(t, u_j)} - e^{-\alpha \cdot \min(t, u_j)}) \right\} \]

• \( \alpha \) mean reversion, measure of speed of convergence to UFR

• \( \zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_N \) parameters to fit actual yield curve

• Kernel functions \( K_j(t) \) - functions of maturity \( t \); depend on input parameters and data from ZC bonds

\[ K_j(t) = W(t, u_j), \ t > 0 \text{ and } j=1, 2, 3...N \]
Extrapolation of yield curve – Solvency II extrapolation method (Smith-Wilson method (3))

- Unknown parameters $\zeta_j, j=1, 2, 3 \ldots N$, are solutions of:

\[
m_1 = P(u_1) = e^{-UFR \cdot u_1} + \sum_{j=1}^{N} \zeta_j \cdot W(u_1, u_j)
\]

\[
m_2 = P(u_2) = e^{-UFR \cdot u_2} + \sum_{j=1}^{N} \zeta_j \cdot W(u_2, u_j)
\]

\[
\ldots
\]

\[
m_N = P(u_N) = e^{-UFR \cdot u_N} + \sum_{j=1}^{N} \zeta_j \cdot W(u_N, u_j)
\]
Extrapolation of yield curve – Solvency II extrapolation method (Smith-Wilson method (4))

i.e.:

\[ m = p = \mu + W \zeta, \]

with:

\[ m = (m_1, m_2, \ldots, m_N)^T, \]
\[ p = (P(u_1), P(u_2), \ldots, P(u_N))^T, \]
\[ \mu = (e^{-UFR \cdot u_1}, e^{-UFR \cdot u_2}, \ldots, e^{-UFR \cdot u_N})^T, \]
\[ \zeta = (\zeta_1, \zeta_2, \ldots, \zeta_N)^T, \]

and

\[ W = (W(u_i, u_j))_{i=1,2,\ldots,N; j=1,2,\ldots,N} \text{ a } N \times N \text{-matrix of certain Wilson functions} \]
Extrapolation of yield curve – Solvency II extrapolation method
(Smith-Wilson method (5))

• Solution \((\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_N)\) found via:

\[
\zeta = W^{-1}(p - \mu) = W^{-1}(m - \mu),
\]

• Then, for all maturities \(t\):

\[
P(t) = e^{-UFR \cdot t} + \sum_{j=1}^{N} \zeta_j \cdot W(t, u_j), \quad t > 0
\]

• Then, calculate spot rates:
  
  \[
  \tilde{R}_t = \frac{1}{t} \cdot \ln\left(\frac{1}{P(t)}\right)
  \]
  
  for continuously compounded

  \[
  R_t = \left(\frac{1}{P(t)}\right)^{\frac{1}{t}} - 1
  \]
  
  for annual compounding
Extrapolation of yield curve – reference to comparable currencies

• May be possible to find comparable/closely related currency where longer market-observable yields exist
• Use forward rates of related currency to extrapolate (other methods possible)
• In example local currency yield extrapolated after 5 years, e.g. for 10 year: 
  \[ \left( 1 + 5\text{yr local} \right)^5 \times \left( 1 + 10\text{yr forward related} \right)^5 \right)^{(1/10)} - 1 \]
Credit and liquidity risks - interrelation

- Market spread of asset over “risk-free” can reflect both credit risk and liquidity risk
- Prior to financial crisis, most of spreads on corporates attributed to credit risk
- During the financial crisis illiquidity spreads accounted for considerable amount of widening of spreads
- Insurer of good credit standing, but assets and liabilities poorly matched (duration 10 and 20 years respectively) => potential forced sale of illiquid assets
- Insurance liabilities not frequently traded => measurement of credit and liquidity risk more difficult
**Liquidity risks**

- Two sides:
  - For policyholder – restrictions in selling or transferring liability.
  - For insurer – need to liquidate assets to meet liability cash-flows
Liquidity risks – estimating liquidity premium of assets

• Methods:
  – Credit Default Swap (CDS) Basis method
  – Structural model method
  – Covered bond spreads method
  – Proxy method
Liquidity risks – estimating liquidity premium of liabilities

- MCEV Principles, Solvency II and IASB Insurance Contracts EB allow for LP to be included in discount rate for liabilities (although not referred to as such in Solvency II any more)
- (Mainly) relate to characteristics of liabilities
- Life insurance liability liquid for insurer if cash-flows highly uncertain (timing and amount):
  - Easy surrender => high uncertainty => high liquidity => low LP
  - No surrender => highest LP (can also be backed by illiquid assets). E.g. single premium fixed annuity
- Consider dynamic policyholder behaviour


**Liquidity risks – estimating liquidity premium of liabilities (2)**

- CEIOPS Task Force report 2010: LP should reflect liabilities - currency, timing, ability to surrender, and ability to pass loss to policyholder from sale of asset forced due to lack of liquidity.

- Others argue: should be based on underlying assets (total credit spread – credit risk – (perhaps) sovereign risk, currency risk etc).

- MCEV Principles: apply where liabilities not liquid; base on market data; allow for constraints (regulatory, internal, investment) on company’s ability to access LP.
  - “A liability is liquid if the liability cashflows are reasonably predictable”

- IASB: LP need only refer to liabilities, not assets.
Liquidity risks – estimating liquidity premium of liabilities (3) – replicating portfolios

- Estimate LP of asset portfolio replicating liabilities
- Some argue basing on actual underlying assets wouldn’t give market value of liabilities
- RP approach has limitations:
  - RP of assets may not exist, or may not have market observable prices
  - May not be able to capture uncertainties in liability cash-flows (eg policyholder behaviour)
  - Where some liability cash-flows depend on performance of underlying assets, RP for all the cash-flows may not be possible (e.g. participating business)
Liquidity risks – estimating liquidity premium of liabilities (4) – replicating portfolios

- Pritchard and Turnbull propose method for estimating predictability of liability cash-flows, based on required disinvestments of RP over stochastic scenarios.
- CEIOPS task-force report estimation of liability LP (varying by maturity and currency):
  \[ F \times G \times LP_{\text{assets}} \]
  - \( F \) – function of maturity of liability and longest maturity for which LP applicable (based on assets in deep, liquid and transparent markets)
  - \( G \) – level of predictability of cash-flows (bucket approach proposed)
  - \( LP_{\text{assets}} \) – estimated LP of assets.
Credit risks

• Credit risk of insurance liability – risk insurer will default on insurance liability when benefits fall due

• Insurer’s credit risk – assessed at entity level; reflects ability to fulfill all obligations (e.g. including bonds, employee pension liabilities). Insurance liabilities are often the most senior debt

The two are not the same

• Credit risk of insurance liability:
  – Reflects exposure of policyholder, affected by insurer’s credit risk
  – Can be caused by financial difficulties of insurer
  – Reflects characteristics of liability
Credit risks – add adjustment to discount rates?

For:

• Consistent with asset valuation
• Can prevent overstating value of liabilities
• For insurer, liability credit risk passed to policyholders, who demand a premium
• Such risks affect the liability cash-flows
• Can prevent day one loss, possible when pricing assumes higher than risk free returns
Credit risks – add adjustment to discount rates?

Against:

• May be counterintuitive:
  – As insurer’s credit standing worsens, liability value falls => stronger financial position
• Insurers valued on going concern basis
• Higher credit risk should increase capital requirements, but not reduce liability
• Higher expected returns reflected in pricing should only be included when actually earned (such pricing could result in higher credit risk)
Credit risks – pension plans

• Pension plan assets for funded plans often in separate vehicle, usually with restrictions on their use by the employer
• However, participants still exposed to credit risk of plan sponsor => financial difficulties can cause plan to become underfunded
• Participants also exposed to credit risk of underlying assets
• In some countries insurance schemes exist, but may not fully eliminate credit risk
Credit risks – own credit risk

- Own risk generally not reflected in discount rates for liabilities (e.g. MCEV Principles, Solvency II, IFRS Phase II, statutory or GAAP accounting in the US)
- In highly regulated market default risk to policyholders very low?
- Credit risk of liabilities may differ from insurer’s issued bonds
- IFRS9 financial instruments – initial measurement at market value, hence allows for company’s own credit risk
- To determine own credit risk could consider insurer’s bonds
Inflation – choice of measure

- Consumer price index (CPI) – based on price of standardized, representative basket of goods and services purchased by average consumer
- CPI may be broken down by sector, and, in some cases further by geographic region and consumer type
- Some measures exclude goods with high price volatility, such as food and energy (e.g. Core CPI in the US)
- Need to consider which components to use in assessing inflation for particular liabilities
Inflation – choice of measure (2)

• Other measures include:
  – Producer price index (PPI), reflecting prices which producers receive
  – GDP deflator, reflecting prices of all new, domestically produced, final goods across the economy
• These measures are different but tend to be highly correlated
• Wage inflation is related, but different to price inflation
Inflation – choice of measure (3)

• Choice of measure for discount rate purposes?
• Where liability not directly related to inflation (e.g. fixed annuity), choice of inflation measure implicit in reference rate
• Where liability directly related to inflation (e.g. index-linked annuity, health benefits, P&C) use most appropriate measure, taking into account, e.g.:
  – Contracts
  – Prior precedents
  – Policyholder communications
  – PRE
Inflation – the Fisher relationship

Nominal yields made up of:

- Real yields, observed in inflation-linked bond market or other sources, derived from historical experience or estimated
- Expected future inflation – relatively subjective
- Inflation risk premium

Break-even inflation:

- Equates returns on inflation-linked and nominal bonds
- Nominal yield – real yield of equal duration
- Cost of hedging inflation risk
- Highly correlated to actual inflation
Inflation – the Fisher relationship (2)

Inflation-risk adjustment:
• Relatively small (average usually <50bp)
• Varies over time
• Typically upward sloping
• Can be influenced by imbalance in supply and demand of bonds

Long-term inflation rates also depend on government and central banks’ fiscal and monetary policies
# Inflation – inflation risk

Includes hyperinflation, deflation, moderately high inflation, inflation spikes

Hyperinflation:
- Caused by printing money not supported by growth in output
- => loss of confidence in currency
- Has occurred many times in history; in recent years mainly emerging economies
- Risk nowadays lower in developed economies

Deflation, some examples:

<table>
<thead>
<tr>
<th>Country</th>
<th>Years</th>
<th>Deflation Total Impact</th>
<th>Deflation % per annum</th>
<th>Event / Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1921 to 1932</td>
<td>-32%</td>
<td>-3.20%</td>
<td>Great Depression</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1920 to 1933</td>
<td>-37%</td>
<td>-3.60%</td>
<td>Great Depression</td>
</tr>
<tr>
<td>Australia</td>
<td>1929 to 1933</td>
<td>-22%</td>
<td>-6.10%</td>
<td>Great Depression</td>
</tr>
<tr>
<td>West Germany</td>
<td>1949 to 1950</td>
<td>-7%</td>
<td>-3.70%</td>
<td>End of World War II</td>
</tr>
<tr>
<td>Japan</td>
<td>1999 to 2009</td>
<td>-4%</td>
<td>-0.40%</td>
<td>Capital market bust</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>1999 to 2004</td>
<td>-15%</td>
<td>-2.60%</td>
<td>Asian financial crisis</td>
</tr>
<tr>
<td>Ireland</td>
<td>2009</td>
<td>-5%</td>
<td>-5%</td>
<td>Credit crisis, banking debts</td>
</tr>
</tbody>
</table>
### Inflation – inflation risk (2)

<table>
<thead>
<tr>
<th>Country / Region</th>
<th>Start Year</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>r. Auto-Correla</th>
<th>Distribution of Annual Inflation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&lt;0%</td>
</tr>
<tr>
<td>US</td>
<td>1990</td>
<td>2.70%</td>
<td>1.20%</td>
<td>-0.12</td>
<td>0%</td>
</tr>
<tr>
<td>UK</td>
<td>1990</td>
<td>2.50%</td>
<td>1.80%</td>
<td>0.75</td>
<td>0%</td>
</tr>
<tr>
<td>Eurozone</td>
<td>1990</td>
<td>2.20%</td>
<td>0.80%</td>
<td>0.5</td>
<td>0%</td>
</tr>
<tr>
<td>US</td>
<td>1950</td>
<td>3.70%</td>
<td>2.90%</td>
<td>0.74</td>
<td>2%</td>
</tr>
<tr>
<td>UK</td>
<td>1950</td>
<td>5.30%</td>
<td>4.80%</td>
<td>0.81</td>
<td>0%</td>
</tr>
<tr>
<td>Eurozone</td>
<td>1950</td>
<td>2.60%</td>
<td>2.10%</td>
<td>0.37</td>
<td>5%</td>
</tr>
<tr>
<td>US</td>
<td>1913</td>
<td>3.20%</td>
<td>5.00%</td>
<td>0.65</td>
<td>11%</td>
</tr>
<tr>
<td>UK</td>
<td>1900</td>
<td>4.10%</td>
<td>6.10%</td>
<td>0.75</td>
<td>11%</td>
</tr>
<tr>
<td>Eurozone</td>
<td>1948</td>
<td>2.50%</td>
<td>2.20%</td>
<td>0.42</td>
<td>6%</td>
</tr>
</tbody>
</table>

Above shows:

- Relatively modest inflation over last 20 years
- Inflation higher and more volatile over longer periods
- Inflation tends to exhibit regime switching => dangerous to consider only recent history
Inflation – models

• Where markets in nominal and index-linked securities are liquid, can measure market expectation of future inflation using Fisher equation
• Where markets illiquid care needed interpreting market data
  – Also the case, e.g. in UK where DB pension funds hold a lot of index-linked gilts
• Other sources of information:
  – Historic and current inflation and nominal yields
  – Central bank target inflation and inflation projections
  – Economists’ surveys of inflation
• However, medium to long term inflation can be impacted by uncertain factors, e.g.:
  – Different objectives of central banks
  – Shifts in the economy between consumption and saving
  – Imported inflation (domestic economy has little control)
• In some cases, real discount rates could be used, e.g. for inflation-adjusted pensions liabilities